

## Time reversal symmetry breaking superconducting states in Cuprates

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**Abstract** — We study the doping and temperature dependence of the order parameter for the time reversal symmetry breaking states  $d_{x^2-y^2} + is$  and  $d_{x^2-y^2} + id_{xy}$  using a tight binding model that includes orthorhombic distortion and second nearest neighbor hopping,  $\gamma$ . Mixing occurs both in square lattice and in the presence of orthorhombic distortion and also for various values of  $\gamma$  when studied as a function of filling. We also study the specific heat for the mixed states and the coherence length for the pure  $d_{x^2-y^2}$  and  $d_{xy}$  waves.

**Keywords** — Order parameter, symmetry breaking, super conducting, cuprates.

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In 1986 superconductivity was discovered in LBCO (an oxide of Lanthanum, Barium and Copper) with critical temperature  $T_c \approx 35$  K initiating the era of high temperature superconductors (HTSC) [1]. In these 13 years the critical temperature has increased considerably to reach nearly four times the limit considered in the conventional superconductors and the most promising ones are those containing an oxide of copper, known as cuprates. The cuprates consist of one or more  $\text{CuO}_2$  planes in their structure separated by layers of other atoms (Ba, La...). Presently two of the leading candidates for technological application of superconductivity are  $\text{LnBa}_2\text{Cu}_3\text{O}_{7-\delta}$  and  $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$ . Most of the cuprates are doped materials. For example, substitution of the divalent  $\text{La}^{2+}$  for  $\text{La}^{3+}$  in the antiferromagnetic insulator  $\text{La}_2\text{CuO}_4$  produces superconductivity in  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  [2].  $T_c$  is also very sensitive to the level of doping. These new materials not only show a high  $T_c$  but also various anomalous normal state properties. The peculiarities of these superconductors have been a challenge for the thousands of scientists working around the globe.

The pairing symmetry provides clue to the identity of the superconducting pairing mechanism which is essential for the development of the theory of HTSC. It still remains a subject of intense debate even after more than a decade of

intense research [3]. Unlike the conventional superconductors these new materials are not  $s$ -wave compounds. Several experiments and theoretical studies suggest a  $d_{x^2-y^2}$  wave pairing in cuprates. The phase sensitive pairing symmetry experiments [4] have given a strong evidence for predominantly anisotropic  $d$ -wave pairing in a number of optimally doped cuprates. However, there have also been considerable theoretical studies [5,6] indicating that a pure  $d_{x^2-y^2}$  is not stable against the formation of time reversal symmetry breaking states such as  $d_{x^2-y^2} + id_{xy}$  or  $d_{x^2-y^2} + is$  below a certain characteristic temperature. Various indirect experimental evidences for the existence of a major  $d$ -wave component with a minor imaginary  $s$  and  $d_{xy}$  component have been reported in cuprates like  $\text{YBa}_2\text{Cu}_3\text{O}_7$  [7] and  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  [6] respectively.

Till today there has been no consensus as to the mechanism causing high  $T_c$  in these materials. So the search for the new materials has been empirical as no predictive theory is currently known. In absence of a microscopic theory we use a phenomenological two-dimensional tight binding model with appropriate lattice symmetry.

We study the two-dimensional single-band tight binding model with electronic dispersion relation including upto second nearest neighbor hopping to investigate the

behavior of the states  $d_{x^2-y^2} + i\chi$  with filling and temperature. Here  $\chi$  can be  $s$  or  $d_{xy}$ . We consider the order parameter dependence on filling and temperature both in square lattice and also in presence of orthorhombic distortion. We also exhibit the temperature dependence of specific heat for the  $d_{x^2-y^2}$  wave and the coherence length of the pure  $d$  waves. The quasiparticle dispersion relation is given by  $\epsilon_k = -2t[\cos k_x + \beta \cos k_y - \gamma \cos k_x \cos k_y - \mu]$ , where  $t$  and  $\beta t$  are the nearest neighbor hopping integrals along the in plane  $a$  and  $b$  axes respectively and  $\gamma t$  is the second nearest neighbor hopping integral.  $\mu$  is the chemical potential. The gap ( $\Delta_k$ ) equation in this model is

$$\Delta_k = - \sum_q V_{kq} \frac{\Delta_q}{2E_q} \tanh \frac{E_q}{2k_B T}, \quad (1)$$

with  $E_q = [(\epsilon_q - E_F)^2 + |\Delta_q|^2]^{1/2}$ , where  $E_F$  is the Fermi energy and  $k_B$  is the Boltzmann constant. The interaction is given by  $V_{kq} = - \sum_i V_i \eta_{ik} \eta_{iq}$  irreducible representation of the point group of square lattice  $C_{4v}$  and are appropriate generalization of the circular harmonics incorporating the proper lattice symmetry. We define  $\eta_{1k} = \cos k_x - \beta \cos k_y$ , for  $d_{x^2-y^2}$  wave, and  $\eta_{2k} = 2 \sin k_x \sin k_y$  (1) for  $d_{xy}$  wave. The order parameter has the following anisotropic form :  $\Delta_k \equiv \Delta_1(\cos q_x - \beta \cos q_y) + i\Delta_2 \eta_{2k}$  where  $\eta_{2k}$  can be for  $s$  or  $d_{xy}$  wave. Using the above forms of  $\Delta_q$  and  $V_{kq}$ , eq. (1) becomes a coupled set of equations, where the coupling is introduced through  $E_q$  [8,9,10]. The filling dependence can be determined from the number equation

$$n = 1 - \sum_q \frac{\epsilon_q - \mu}{E_q} \tanh \frac{E_q}{2k_B T}. \quad (2)$$

We also study the specific heat  $C_s(T)$  and coherence length  $\xi^2$ .

We solve the coupled set of gap equations [8,9] along with (2) numerically and calculate  $\Delta_1$  and  $\Delta_2$  at various  $n$  and temperatures. We perform calculations on a perfect square lattice ( $\beta = 1$ ) for both  $d_{x^2-y^2} + i\chi$  mixing with  $V_1 = 1.7t$ , and  $V_2 = 0.73t$ . In the presence of an orthorhombic distortion ( $\beta = .95$ ) the potentials are  $V_1 = 2.1t$ , and  $V_2 = 0.97t$ .

In Figure 1 we plot  $\Delta_{x^2-y^2} = \Delta_1$  and  $\Delta_{xy} = \Delta_2$  for  $d_{x^2-y^2} + id_{xy}$  with  $\gamma = 0$  and  $0.1$ . Mixing is present in all the above cases with  $d_{x^2-y^2}$  as the dominant component. As  $n$  decreases we observe a rapid fall of the  $d_{x^2-y^2}$  wave leading only to the  $d_{xy}$  wave. We observe a pronounced maximum in the  $d_{x^2-y^2}$  wave accompanied by a minimum in the  $d_{xy}$  wave. Moreover, the maximum shifts towards a lower value of  $n$  as we increase  $\gamma$ . Unlike the  $\beta = 1$  case, we observe a dip near the maximum along with a rise in the minimum for  $\beta = .95$  case.

In Figure 2 we plot  $\Delta_1$  and  $\Delta = \Delta_2$  in case of  $d_{x^2-y^2} + is$  wave for the same parameters as in Figure 1 [9]. We observe similar behavior as in the previous case, except that the variation is much sharper.  $d_{xy}$  wave falls faster than  $s$  wave with  $n$  and goes to zero much earlier.

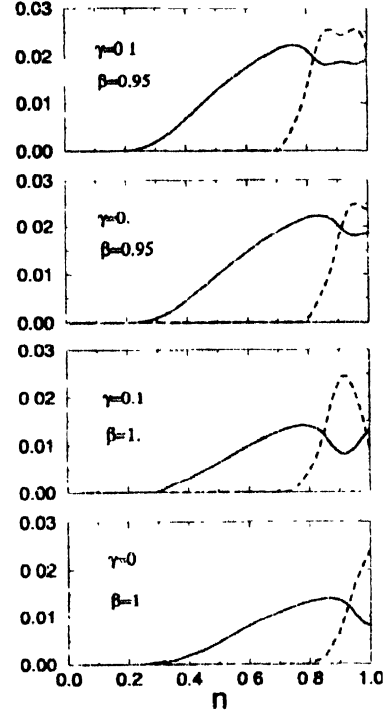


Figure 1. The order parameters  $\Delta_{x^2-y^2}$  (dashed line),  $\Delta_{xy}$  (full line) ( $d_{x^2-y^2} + id_{xy}$ )-wave at different filling.

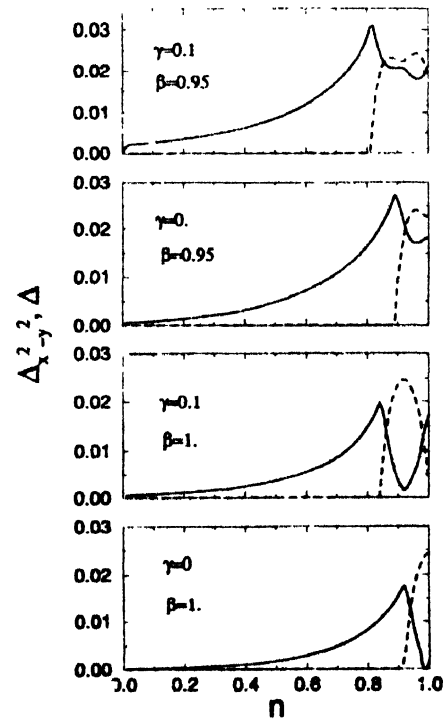


Figure 2. The order parameters  $\Delta_{x^2-y^2}$  (dashed line),  $\Delta$  (full line) for ( $d_{x^2-y^2} + is$ )-wave at different filling.

In Figure 3 we plot the temperature dependence of the specific heat for the normal state and for  $d_{x^2-y^2} + is$  states with  $V_2 = .73t$  and two values of  $V_0$ : i)  $V_0 = 1.92t$  and ii)  $V_0 = 1.8t$ . The specific heat shows two jumps one at  $T_c$  and the other at  $T_{c1}$  ( $\Delta_2 = 0$ ). Hence we observe a second phase transition at  $T_{c1}$  [8,9,11]. For  $T < T_{c1}$  the temperature dependence of the mixed wave specific heat is exponential  $s$ -wave type. Above  $T_c$  the curve follows the power-law behavior of the pure  $d_{x^2-y^2}$ . In  $d_{x^2-y^2} + id_{xy}$  state we observe similar behavior [8].

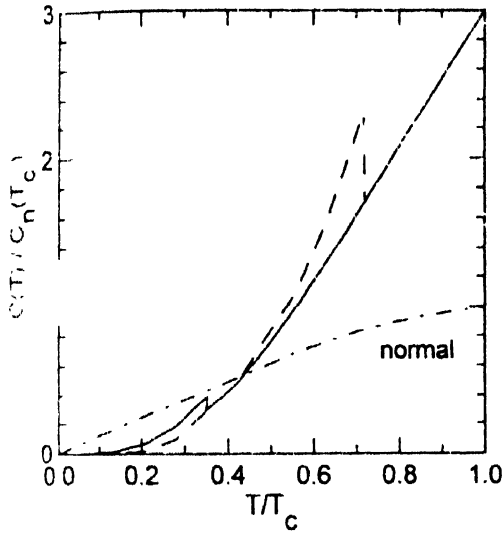


Figure 3. Specific heat ratio  $C(T)/C_n(T_c)$  vs  $T/T_c$  for  $(d_{x^2-y^2} + is)$  for models i) (dashed line) ii) (full line) and the normal state (dashed-dotted line)

In Figures 4 and 5 we plot the coherence length as a function of filling for pure  $d_{x^2-y^2}$  and  $d_{xy}$  states. For higher values of  $n$ ,  $\xi^2$  approximately remains constant but for high doping we find a steady increase in its value [12].

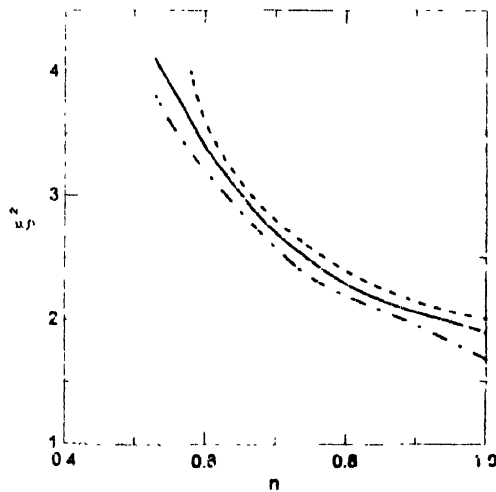


Figure 4.  $\xi^2$  vs  $n$  for  $d_{x^2-y^2}$  wave.  $\gamma = 0$  (dashed line),  $\gamma = .1$  (full line) and  $\gamma = 2$  (dashed-dotted line).

We also observe a decrease in the value of  $\xi$  with a increase in  $\gamma$  for constant  $n$ . It is also in accord with the experimental values.

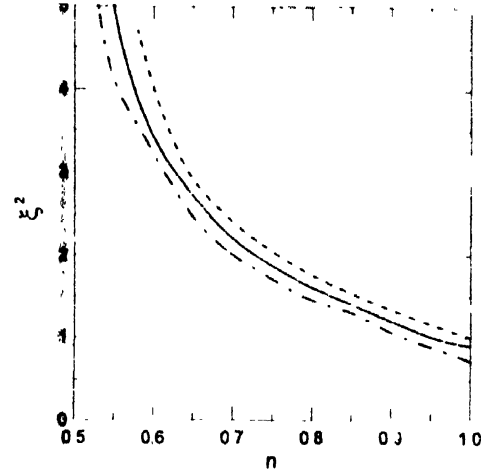


Figure 5.  $\xi^2$  vs  $n$  for  $d_{xy}$  wave.  $\gamma = 0$  (dashed line),  $\gamma = .1$  (full line) and  $\gamma = 2$  (dashed-dotted line)

In conclusion, we have studied the  $(d_{x^2-y^2} + i\chi)$ -wave superconductivity employing a two-dimensional tight binding model on square lattice and also for orthorhombic distortion. We observe that both  $d_{xy}$  and  $s$  wave mix with the  $d_{x^2-y^2}$  wave in square lattice and that with orthorhombic distortion independent of the value of  $\gamma$ . At low doping we have the system is mainly dominated by  $d_{x^2-y^2}$  wave. We also observe a second phase transition at  $T_{c1}$  from Figure 3. We thank FAPESP for financial support.

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